B.Sc. - I (NEP) Semester-I

BSCMA501 / MMCS101 Mathematics DSC : Differential Calculus

P. Pages : 3 **GUG/S/25/15924**

Time : Three Hours Max. Marks : 80

Notes : 1. Solve all the **five** questions.
2. First four questions carry equal **15** marks and the fifth question carry **20** marks.

UNIT - I

1. Solve **any three.**

a) If $f(x)=x^2$, then prove that $\lim\limits_{x\to 3} f(x)=9$ by $\in-\delta$ technique. **5**

b) If $f(x)$ is differentiable at $x=x_0$ then prove that it is continuous at $x=x_0$. **5**

c) Verify the Rolle's theorem for the function $f(x)=(x-a)^m(x-b)^n$, m and n are positive integers and $x\in[a,b]$. **5**

d) Verify the Lagrange's mean value theorem for the function, $f(x)=2x^2-7x+10$ in $[2,5]$. **5**

e) If f and g are continuous real functions on [a, b] which are differentiable in (a, b) then prove that there is a point $c\in(a,b)$ such that $\dfrac{f(b)-f(a)}{g(b)-g(a)}=\dfrac{f'(c)}{g'(c)}$ where $g(b)\neq g(a)$ and $f'(x), g'(x)$ are not simultaneously zero. **5**

UNIT - II

2. Solve **any three.**

a) Obtain Maclaurin's series for $f(x)=\cos x$. **5**

b) Show that $\log(x+h)=\log h+\dfrac{x}{h}-\dfrac{x^2}{2h^2}+\dfrac{x^3}{3h^3}-.....$ **5**

c) Expand $f(x)=2x^3+7x^2+x-1$ in powers of $(x-2)$. **5**

d) Using $\in-\delta$ definition of limit of a function, prove that- $\lim\limits_{(x,y)\to(4,-1)}(3x-2y)=14$ **5**

e) Prove that the function $f(x,y)=x+y$ is continuous, $\forall(x,y)\in R^2$. **5**

UNIT - III

3. Solve **any three.**

a) If $y = \sin(ax+b)$ then prove that $y_n = a^n \sin\left(ax + b + \frac{n\pi}{2}\right)$ and hence find y_8, for $y = \sin(1-x)$. **5**

b) If $y = e^{ax}$ sin bx, prove that $y_2 - 2ay_1 + \left(a^2 + b^2\right)y = 0$. **5**

c) If $y = a\cos(\log x) + b\sin(\log x)$ then show that $x^2y_2 + xy_1 + y = 0$ and $x^2y_{n+2} + (2n+1)xy_{n+1} + \left(n^2+1\right)y_n = 0$. **5**

d) If $y = \left(x^2 - 1\right)^n$ then prove that $\left(x^2-1\right)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$. **5**

e) If $y^{1/m} + y^{-1/m} = 2x$, then prove that- **5**
$\left(x^2-1\right)y_{n+2} + (2n+1)xy_{n+1} + \left(n^2 - m^2\right)y_n = 0$.

UNIT - IV

4. Solve **any three.**

a) If $u = \dfrac{e^{x+y+z}}{e^x + e^y + e^z}$ then show that $u_x + u_y + u_z = 2u$. **5**

b) If $u = F(x-y, y-z, z-x)$ then prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$. **5**

c) If $u = f(x, y)$ be a homogeneous function of x, y of degree n then prove that **5**
$x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = n(n-1)u$.

d) Expand e^{xy} at the point (2, 1) up to first three terms. **5**

e) Find the maximum and minimum value of $x^3 + y^3 - 3axy$. **5**

5. Solve **any ten.**

a) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{\sqrt{9-x} - 3}{x}\right)$ **2**

b) Define a continuous function at $x = x_0$. **2**

c) State the Lagrange's mean value theorem. **2**

d) Write the Maclaurin's series for f(x). **2**

e) Write the $\in - \delta$ definition of limit of function of two variables. **2**

f) Using algebra of limits, prove that- **2**

$$\lim_{(x,y)\to(0,2)} \left\{ \sin x + x \tan y - \frac{\cos^{-1} x}{y^2} + 2y^2 \right\} = 8 - \frac{\pi}{8}$$

g) Find y_3, if $f(x) = (2x-3)^4$. **2**

h) If $y = A \sin mx + B \cos mx$ then prove that $y_2 + m^2 y = 0$. **2**

i) Find y_n, for $y = x^2 e^{ax}$. **2**

j) If $z = f(x, y)$, show that $xz_x - yz_y = 0$. **2**

k) If $x = r\cos\theta, y = r\sin\theta$, find $\dfrac{\partial(x, y)}{\partial(r, \theta)}$ **2**

l) Define stationary point of f (x, y). **2**

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